Supporting Independent Thinking Through Mathematical Education

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Introduction

Mathematical education is recognized internationally as central to society. The teaching of mathematics begins at a young age, because basic mathematical concepts are at the heart of both personal and social development. There is no doubt that issues connected with mathematics education need to be at the center of attention of political leaders, educationalists the general community, and, of course, parents and teachers.

Teaching mathematics is important both for supporting the development of the child and for solving critical problems in a global society. Number sense, numerical literacy, spatial abilities and other fundamental skills and concepts of mathematics, are critical to social and personal growth and understanding. By means of it, mathematical knowledge gained with the help of teachers can favour the pupils with logical thinking and reasoning, which aids the conduct of dialogue and negotiation. In this way, mathematics supports ethical behavior, especially understanding human rights and obligations. The ability to organize and use data is valuable in almost all spheres of individual and social life. The search for solutions promotes creativity, flexibility, and adaptation to new situations, and succes in finding (multiple) solutions supports the development of self-esteem.

The quality of teaching and learning mathematics depends on many elements, affected and determined by each other. While many factors, such as social structures of inequity and diversity, are seemingly beyond the remit of the individual teacher, he or she remains a central element, responsible for what is going on during lessons in their classroom. Teachers must understand their role, both within the classroom, and as a part of larger social and political structures. They must blend their interactions with pupils and their understanding of mathematical content objectives with their own ethical and moral commitments in order to effect change in society.

Teacher-training in mathematics goes far beyond subject-specific and pedagogical content. It connects with many other realms: psychology (creation of concepts, emotions, motivations, interactions, ...), linguistics (communications, language in learning and teaching mathematics, symbol creation and its understanding, ...), socio-cultural theory (ethno-mathematics, equity and diversity, ...), history and epistemology (developments of mathematical concepts, historical obstacles in understanding mathematical concepts, ...), technology (application of technology in mathematics, using computers in teaching mathematics, ...), and so on.

Few people enter the field of teaching with a comprehension of the complexity that such work entails. The education of the teachers of the future, and the ongoing professional development of practicing teachers must help them to negotiate these complexities and to reconcile the potential conflicts between the realities of teaching and their own personal moral and professional commitments.

Open questions in mathematical education

Part 1

Tatyana Oleinik
Nikolay Ivashenko
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Ukraine

Problems of formation learning environment

We discuss on major problems and ways of Ukrainian pedagogical innovation which based on democratic transformations in education. Our special study is formation learning community (environment) of successful learners. That modifying methodical frameworks focus on providing student's capacity to be engaged in cooperative learning as a capable individual that know how to initiate and create independent opinions, negotiate and build consensus for problem solving and risk-taking. We present a set of the modern methods and innovative leaning strategies in classrooms that increase the students' capacity to form learning environment as inquiry one. We focus on transforming classroom practices so that they provide a climate of trust, engage students in interesting activity and foster deep inquiry and genuine debate.

The realization of this approach is based on the idea that successful learning activities assume some features of inquiry; a lesson begins to resemble a project with meaningful classroom dialogue and inquiry [1]. It involves taking ideas and examining their implications, exposing them to polite skepticism, balancing them against opposing points of view, constructing supporting belief systems to substantiate them and taking a stand based on those structures. We have convinced that environment stimulates purposeful and productive activities, not traditionally „study work": students are engaged in the practical intellectual work of finding solutions to problems that originate from the real world. It improves the educational process that enables the students to acquire the mathematical knowledge more firmly, to form the practical abilities to use them.

We pay attention to the main peculiarities of learning environment and conditions of one's realization:

(i) goals of education: they reflect the students' hopes based on dialogue (the productive exchange of ideas, attitudes such as tolerance, careful listening to others, assuming responsibility for one's own positions and so on);
(ii) a role of the pedagogue: it reconstructs reality in a problematic form, with the students perceiving and analyzing this reality, the curriculum assumes that students' interests are to be taken into account;
(iii) subject: a teacher (active) and a student (active), object: the entire surrounding world;
(iv) the knowledge is subject to doubt: the doubt must stimulate dialogue, a critical approach and creative activity, education is a creative task;
(v) drastic change of reality according to human needs;
(vi) existence of a problem stimulates the search for its solution.

Three points advance students in their intellectual development [2]:

(i) they should be faced with choices, with materials that invite their comfortable and familiar ways of considering things, get more than one interpretation and the challenge to make and defend their own interpretations;

(ii) they should hear their classmates express points of view different from their own;

(iii) they should be encouraged to reflect, especially in writing, on the ways in which their thinking is changing.

We pay attention to the meta-cognitive processes, i.e. mastering of "thinking strategies", "implementation rules" for cognitive activity. In this context it is essential to overcome stereotypes like "right" and "wrong" responses. Formation the learning environment is impossible without usable knowledge as well as knowledge concerning essence of inquiry and different ways of one. The students create more knowledge and solutions to practice problems, but also to be able to the systematic and efficient habits of idea creation based on the knowledge of the key discipline concepts.

The students know the valuable thing of collaborative/cooperative work and the improvement of their argumentation during conducting research and presenting results to others. In the learning environment the students are attracted to listening to different opinions of their classmates and creating an atmosphere that support free acceptance of the another ideas or argumentative rejection them, tolerant and interdependent social behavior.

In the formation of learning environment the teacher's role is facilitating students, that realized by three objective: to design contexts that promote one's inquire for thinking; to develop strategies for encouraging thinking skills; to develop strategies, especially portfolio, for authentic assessment of thinking as a mean of evaluating work. The main results of that approach is changing students' role, that realized in self-searching activities and grouping forces (instead of remembering ready-made knowledge). Moreover it makes available shifting their attention to the individual peculiarities and uniqueness that provides possibilities of solving educational problems as individually as in group; encouraging their interests to following and training methods of inquiry (investigating, experimenting, interviewing, surveying, writing, and so on).

There are some work questions [2]: What is the main question posed by this piece? What answer does it offer? What reasons are offered in support of that answer? What evidence is offered in support of each reason? What reasons or facts are left out – things that might have supported different answer to the question? What "facts" are we expected to accept on faith? What nominal assumptions are made? What value assumptions are made?

References

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2. Temple, Ch. et al.: 2001, Critical Thinking for Universities: Guidebook, Kyiv.